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The Formation of Sunspots from the Solar Toroidal Field

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The Formation of Sunspots from the Solar Toroidal Field

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Abstract

It is shown that a horizontal magnetic flux tube in an electrically conducting atmosphere is bouyant and will tend to rise. This magnetic bouyancy is large enough to bring an occasional strand of flux from the general solar toroidal field up into the photosphere if we assume general field densities of 100 gauss farther down. Identifying the intersection of such ropes with the photosphere as the source of sunspots, several general characteristics of spots may be deduced, e.g. east-west orientation, bipolarity, appearance only in low latitudes, migration, reversal of polarity, etc.

The static equilibrium equations for a flux tube are developed. With a cooling mechanism, such as suggested by Biermann (1941), we find from the equilibrium equations that a sunspot group should consist of a diffuse flux tube of 10 to 100 gauss and 10^5 km extent in the photosphere forming eventually a number of cool intense cores of several thousand gauss.

Sources of prominence motions are treated in the last section. The most violent motions obtainable are of the order of 100 km/sec and are associated with sunspots, suggesting surge prominences.

The Formation of Sunspots from the Solar Toroidal Field

1. Introduction

The generation of magnetic fields within the sun has been discussed in a preceding paper (Parker 1954). There it was shown that we should expect dynamo waves just under the surface of the sun in the convective zone migrating from the polar to the equatorial regions. The waves are prevented from diffusing out of the convective zone by the high conductivity of the medium. The waves consist in part of bands of toroidal magnetic field, the sense of the field alternates from one band to the next. The poloidal field is $\pi/2$ out of phase with the toroidal field, essentially occupying the regions between the intense toroidal bands. This is shown schematically in figure 1. Observation of secondary magnetic phenomena such as sunspots indicates that there are two (or at most three) toroidal bands in each hemisphere at any one time, and that about 22 (or 33) years is required for the migration of each band from the pole to the equator. Dynamical considerations indicate that the initial amplification of the relatively weak wave starting at the pole is primarily of the poloidal components; by the time middle latitudes are reached the decrease of cyclonic motions and the increase of the nonuniform rotation of the sun shift the amplification to the toroidal field. Thus from the pole to the middle latitudes the poloidal component of the traveling wave predominates; in low latitudes the toroidal field predominates until both the poloidal and toroidal fields

finally vanish at the magnetic equator.

The problem before us now is the question of what secondary magnetic effects might be expected around the fringes of the solar dynamo. We have in mind of course the obviously magnetic phenomena such as sunspots and prominences, as well as those occurrences such as spicules, flares, etc. which one suspects must be of magnetic origin because no purely hydrodynamical explanation seems to exist.

In this paper we discuss what we shall call magnetic bouyancy: Consider a magnetic flux tube running horizontally through a gaseous electrically conducting medium such as one finds in the sun. It is well known that the tensile stress in the tube is $B^2/(2\mu)$ in mks units, where B is the magnetic field density. The magnetic field also exerts an outward pressure, and, were the tube not impeded by the surrounding matter, it would expand. As it is, B satisfies the diffusion equation

$$\partial \underline{B} / \partial t = \nu_m \nabla^2 \underline{B}$$

where ν_m is the magnetic viscosity. If the medium is a sufficiently good conductor, ν_m becomes small enough that

$$\partial \underline{B} / \partial t \cong 0$$

and the field does not diffuse through the medium. Hydrostatic equilibrium requires that the magnetic pressure p_m be balanced by the gas pressure p_g outside the tube. Thus, if p_i is the gas pressure inside the tube, we must have

$$p_o = p_i + p_m \tag{1}$$

Now

$$p_i = B^2 / (2\mu) \quad (2)$$

and is always positive. Thus $p_i < p_e$. Supposing that the temperature of the gas within the flux tube is the same as the temperature outside, we are led to the conclusion that $\rho_i < \rho_e$. Thus, the flux tube is in effect a bubble and will try to rise: this is the magnetic bouyancy referred to above.

The bouyant force per unit length of a tube of cross-sectional area A is $g(\rho_e - \rho_i)A$. The tension is $A B^2 / (2\mu)$. Consider a length L of the flux tube clamped at both ends. If the tube is to be able to rise, we must require that the bouyant forces exceed the tension at the ends of the length, which will try to hold the length in place. Thus we must satisfy an approximate relation of the form

$$L g (\rho_e - \rho_i) A > 2 A B^2 / (2\mu) \quad (3)$$

Now

$$p = k T \rho / m \quad (4)$$

where k is Boltzmann's constant and m the mass of an individual gas molecule. Using (4) for p_e and p_i , (1) may be rewritten as

$$\rho_e = \rho_i + \left(\frac{m}{kT} \right) \left(\frac{B^2}{2\mu} \right). \quad (5)$$

(3) becomes

$$L > 2 k T / (m g) \quad (6)$$

Thus magnetic bouyancy is effective over any length of flux tube exceeding twice the scale height of the medium.

It should be emphasized that magnetic bouyancy is not an instability in the usual sense: The bouyant force per unit

volume is the quantity

$$F_b = \left(\frac{mg}{kT} \right) \left(\frac{B^2}{2\mu} \right) \quad (7)$$

and a long horizontal flux tube can never be in static equilibrium. So long as F_b is large enough not to be overwhelmed by other motions such as convection and turbulence, the tube will rise.

Conditions within a flux tube that has undergone vertical displacement are investigated at some length in Appendices II and III; it is found that raising a length of a long flux tube results in a flow of fluid along the tube which enhances the magnetic buoyancy in the raised portion. Thus, once the tube has begun to rise, it will not in general stop.

To obtain a quantitative estimate of the buoyancy force, consider a flux tube of 100 gauss at a depth of 2×10^4 km in the sun. At this level $\rho_e \cong 2.5 \times 10^{-4}$ gm/cm³, $T_e \cong 2.5 \times 10^5$ °K. (5) gives $\rho_e - \rho_i \cong 2 \times 10^{-11}$ gm/cm³, which is only 10^{-7} of the density ρ_e . A temperature variation of 0.02 °K would produce the same fluctuation in the density. We see, then, that magnetic buoyancy will be negligible for the general solar field. Consider, however, a relatively intense strand of field of, say, 10^3 gauss produced by an abrupt shearing in the turbulent convective motions at a depth of only 10^3 km. Now $\rho_e \cong 0.9 \times 10^{-6}$ gm/cm³ and $T_e \cong 1.5 \times 10^4$ °K; (5) gives $\rho_e - \rho_i \cong 3 \times 10^{-8}$ gm/cm³. $\rho_e - \rho_i$ is now 0.04 ρ_e and is equivalent to heating the region by 600°K; if the rope is not swept back down into the convective zone by some violent convective flow, it will rise to the surface of the sun.

In the sun, then, we expect to find an occasional strand

from the toroidal or poloidal fields bobbing up to the surface of the sun; the main field will be essentially unaffected. We expect these strands to come up where the bouyant force F_b is strongest and can overcome the random velocity and magnetic fields present in the convective zone. Thus strands of the toroidal field are expected to appear only below the middle latitudes. This leads us to a suggestion by Elsasser* that sunspots seem most naturally explained as a portion of the toroidal field which has been heaved up to the surface of the sun by some dynamical mechanism. The mechanism here assumed is magnetic bouyancy supplemented to an unknown extent by the convective forces existing in the convective zone. Strands of the poloidal field may appear much nearer the pole than strands of the toroidal field because, as was pointed out earlier, the poloidal field is amplified at higher latitudes than the toroidal field. Ropes of flux floating up from the poloidal field may be responsible for the prominence activity observed (Menzel, 1953) in the middle latitudes shortly before the onset of a new sunspot cycle.

If we identify sunspots with the strands of the toroidal field breaking through the photosphere, several of the general properties of sunspots follow immediately: The bipolar character of the spots is due to the two passages of the flux tube through the photosphere, one exit and one entrance; the near east-west orientation of an individual group of spots results from the initial east-west direction of the flux tube; the appearance of the spots only below middle latitudes is due to the fact that the

* Unpublished

toroidal field is not intensely amplified until low latitudes are reached; the migration toward the equator of the region of spot formation is a result of the migration of the underlying toroidal field; the reversal of polarity every half cycle occurs as a consequence of the alternation of sign of successive bands of toroidal field.

2. Formation of a Sunspot

Having shown that several of the general characteristics of sunspots follow from the assumption that the magnetic bouyancy occasionally brings a strand of the solar toroidal field up to the surface, let us now investigate quantitatively the configuration of such a strand upon reaching the surface.

The relatively long life of a sunspot group suggests that the flux producing each group is near static equilibrium. Consider now a vertical flux tube (along the z-axis), it being assumed that after the tube has broken through the surface the part located in the upper convective zone has a sufficiently steep inclination to be considered as vertical. We shall also assume that to begin with the tube does not taper off rapidly. Let the field B be homogeneous across the tube. We shall denote the state of the gas inside the flux tube by ρ_i, ρ_i , and T_i ; outside by ρ_e, ρ_e , and T_e . For static equilibrium of the gas within the flux tube, we must satisfy (1). In addition we must now require that the net force in the z direction be zero or at least approximately so. Thus, in mks units

$$0 = -dp_i/dz - g\rho_i + (1/\mu)[(\nabla \times \mathbf{B}) \times \mathbf{B}]_z. \quad (8)$$

Now

$$[(\nabla \times \mathbf{B}) \times \mathbf{B}]_z = [\partial B_z/\partial y - \partial B_y/\partial z] B_z - [\partial B_z/\partial z - \partial B_z/\partial z] B_z. \quad (9)$$

Our assumption that B be homogeneous across the tube means that

$\partial B_z/\partial y = \partial B_z/\partial z = C$, and (9) reduces to

$$[(\nabla \times \mathbf{B}) \times \mathbf{B}]_z = -(\partial/\partial z)(B_z^2 + B_y^2)/2 \quad (10)$$

Hence (8) becomes

$$dp_i/dz = -g\rho_i - (\partial/\partial z)(B_x^2 + B_y^2)/(2u) \quad (11)$$

We assume that the gas outside the tube is in equilibrium so that

$$dp_e/dz = -g\rho_e \quad (12)$$

Differentiating (1) with respect to z and using (11) and (12), we obtain

$$\frac{\partial}{\partial z} \left(\frac{B^2}{2u} \right) = g(\rho_i - \rho_e) + \frac{\partial}{\partial z} \left(\frac{B_x^2 + B_y^2}{2u} \right)$$

or

$$\frac{\partial}{\partial z} \left(\frac{B^2}{2u} \right) = g(\rho_i - \rho_e). \quad (13)$$

(13) is the equation for longitudinal equilibrium of a vertical flux tube; it states that any change in the longitudinal magnetic stress must be balanced by the local buoyancy.

The assumption of a slowly tapering flux tube implies that $B_x, B_y \ll B_z$, so that, if B is the magnitude of \mathbf{B} , (11) and (13) may be rewritten

$$\frac{dp_i}{dz} = -g\rho_i + O^1(B_x, B_y), \quad \frac{\partial}{\partial z} \left(\frac{B^2}{2u} \right) = g(\rho_i - \rho_e) + O^1(B_x, B_y) \quad (14)$$

Equation (14) applies to an oblique as well as to a vertical flux tube as is shown more generally in Appendix I.

Let us use the static equilibrium equation (14) to investigate the portion of the flux tube rising up from the convective zone through the photosphere. Assume that as a consequence of the slowness of the rise up to the photosphere the tube is in thermal equilibrium with its surroundings, $T_i = T_e$. The sunspot which ultimately results from the flux tube is independent of whether $T_i = T_e$ initially, and so we shall not investigate the

rate of rise and of radiative transfer to see if the assumption is entirely justified; observation indicates that it is. Given that $T_i = T_e$, (1) may be rewritten to give

$$\rho_e - \rho_i = \left(\frac{m}{kT_e}\right) \left(\frac{B^2}{2\mu}\right) \quad (15)$$

where m is the mass of a gas molecule in grams and k is Boltzmann's constant. Using (15), (14) becomes

$$\frac{\partial}{\partial z} \left(\frac{B^2}{2\mu}\right) = -\frac{mg}{kT_e} \left(\frac{B^2}{2\mu}\right) \quad \text{or} \quad \frac{\partial B}{\partial z} = -\frac{mg}{2kT_e} B$$

Integrating, we have

$$B(z) = B(0) \exp \left[-\int_0^z dz \frac{mg}{kT_e} \right] \quad (16)$$

We see that the magnetic field decreases with height with a characteristic length of twice the scale height of the atmosphere. In other words, $B \propto \rho_e^{1/2}$. The width w of the flux tube varies as $\rho_e^{-1/4}$.

ρ_e decreases by one or two powers of ten between the base of the vertical flux tube in the convective zone and the upper end of the tube in the photosphere. Thus, if the flux tube had an initial field density of 100 or 1000 gauss before rising up to the photosphere, $B \propto \rho_e^{1/2}$ implies that we will find fields of only 10 or 100 gauss at the photosphere. This diffuse field will appear over a region of the order of 10^5 km. The configuration of the field is illustrated in figure 2(a); the line PP' represents the level of the photosphere.

The diameter of the flux tube at the photosphere determines how close together the exit and re-entrance of the tube may be; we expect the halves of the bipolar pair forming a sunspot group to be separated by a distance of the order of 10^5 km.

At this stage of development of the flux tube we introduce a cooling postulate (Biermann, 1941; Kuiper, 1953). We assume that the presence of a magnetic field of the order of 100 gauss or more produces a cooling of the region occupied by the flux tube. Biermann's assumption that the cooling follows as a result of the magnetic field's inhibiting effect on convection seems the most straight forward explanation, though our conclusions do not depend critically on details.

We find that the field intensity at the photosphere in a flux tube is remarkably sensitive to the temperature difference between the inside and the outside of the tube: Consider a vertical flux tube in static and thermal equilibrium with its surroundings. Suppose that at the base of the flux tube the field intensity is 100 gauss and that $\rho_e = 10^{-5} \text{ gm/cm}^3$, $T_e = 4 \times 10^4 \text{ }^\circ\text{K}$. Then from (1) we find that $(\rho_e - \rho_i)/\rho_e = 10^{-5}$ and $\rho_e - \rho_i = 10^{-10} \text{ gm/cm}^3$. The important fact is that $\rho_e - \rho_i$ is a very small quantity. If we decrease the temperature T_i inside the flux tube by 1°K , about 2 parts in 10^5 , then ρ_i , in order to maintain the static equilibrium condition (1), must increase by 2 parts in 10^5 . $\rho_e - \rho_i$ changes from $+10^{-10} \text{ gm/cm}^3$ to $-10^{-10} \text{ gm/cm}^3$; from (21) we see that $(\partial/\partial z)[B^2/(2\mu)]$ also reverses its sign without changing its magnitude. Thus, instead of the divergence of the flux tube with height, as indicated in (16) where we find that $B_e \propto \rho_e^{1/2}$, the cooling by 1°K of the interior of the tube results in the tube converging with height and $B \propto \rho_e^{-1/2}$. Hence, a slight cooling effect in a vertical flux tube can result in a tremendous increase in field intensity at the upper end of the tube as shown in figure 2(b); this change of the static equilibrium configuration

of the tube involves a large change in the volume of the tube, as may be seen by comparing figures 2(a) and 2(b). The decrease of volume results in considerable flow of gas along the tube and is discussed in section 4.

To investigate the matter a little further we write (1) in terms of temperature and density. The resulting expression may be rearranged to

$$\rho_e - \rho_i = \frac{\rho_e}{T_i} (T_e - T_i) - \frac{m}{k T_i} \left(\frac{B^2}{2\mu} \right) \quad (17)$$

For vertical equilibrium we put (17) into (14) and obtain

$$\frac{d}{dz} \left(\frac{B^2}{2\mu} \right) = \frac{\rho_e g}{T_i} (T_e - T_i) - \frac{m g}{k T_i} \left(\frac{B^2}{2\mu} \right) \quad (18)$$

As a first approximation let us assume that the cooling effect $(T_e - T_i)$ is simply proportional to the magnetic stresses. Then we write

$$T_e - T_i = \eta [B^2 / (2\mu)] \quad (19)$$

where η is a constant. (18) becomes

$$\frac{d}{dz} \left(\frac{B^2}{2\mu} \right) = \left(\frac{B^2}{2\mu} \right) \frac{m g}{k T_i} \left[\frac{k \rho_e \eta}{m} - 1 \right]. \quad (20)$$

Upon integration we obtain

$$\frac{B^2}{2\mu} = \left(\frac{B^2}{2\mu} \right)_0 \exp \left[\int_0^z \frac{m g}{k T_i} \left(\frac{k \rho_e \eta}{m} - 1 \right) dz \right] \quad (21)$$

$(k \rho_e \eta / m - 1)$ is integrated over distances several times the scale height $k T_i / (m g)$ and appears in the exponent. Thus fields of the order of 2000 gauss are easily obtained from a field of 100 gauss at the base of the flux tube even though $(k \rho_e \eta / m - 1)$ may be only slightly greater than zero. We do not need the intense fields throughout the outer layers of the sun that have been postulated by Gurvich and Lebedinsky (1946).

In conclusion, then, we see that our calculations from the local lateral equilibrium equation (1) and the longitudinal equilibrium equation (14) have shown that the cooling and the intense magnetic field of a sunspot are mutually dependent; before cooling becomes effective we have at the level of the photosphere a diffuse flux tube of 10 to 100 gauss over 10^5 km diameter. Given a cooling effect, however, the diffuse tube forms a dense core. In our greatly oversimplified model, the density of the core increases up to the level where the cooling is no longer effective; at higher levels $T_i = T_e$, and the flux tube diverges according to (16), as is discussed in the next section. We need not assume that the entire cross section of the flux tube goes into the core, because there is undoubtedly a transition region near the surface of the flux tube where the cooling is not very effective. Thus we may expect a spot to be surrounded by a region of diffuse field.

3. The Evolution of a Spot Group.

Consider now the field over a sunspot. If Biermann's mechanism is correct, we expect no cooling because there is no convection above the photosphere; observation indicates that the gas in the chromosphere and the corona over a sunspot is at least not cooler than gas at the same level elsewhere in the solar atmosphere. Thus we are led back to $T_i = T_e$ and the resulting divergence with height (16), of a flux tube in equilibrium. The diameter of such a tube increases with height as $\rho^{-1/4}$. Thus in a region $T = 6000^\circ\text{K}$, yielding a scale height of 200 km, the flux tube increases its width by a factor of e every 800 km; a spot with a diameter of 2×10^4 km at a height z will have a diameter of 5.5×10^4 km at $z + 800$ km. The walls of the flux tube will make an angle of only 1.7° with the horizontal. It must be remembered that this result is only approximate because it was computed from (16) which was derived assuming that the rate of divergence is small. But it serves to show how rapidly the tube is diverging and how the horizontal velocities of the Evershed effect may indeed be gas flowing along the lines of force rather than across them.

The rapid divergence of the tube has interesting dynamical implications. Consider a tube which tapers off abruptly as shown in figure 3(a). The lines of force have no tendency to stick together, and, due to the tension along the lines, will try to separate into several branches at the restriction as shown in figure 3(b).

The tendency for such a breakup is readily demonstrated

by showing that the energy of the field is less after breakup has occurred than before. It will be sufficient to consider the two-dimensional periodic field given by

$$\left. \begin{aligned} B_x &= 0, \quad B_y = [2B_0/(ak)] \sin ky \exp(-z^2/a^2) \\ B_z &= B_0 + B_1 \cos ky \exp(-z^2/a^2) \end{aligned} \right\} \quad (22)$$

This represents an initial uniform field B_0 extending in the z direction. The periodic disturbing field, characterized by B_1 , has the effect of bunching the initial field into bundles at intervals of $2\pi/k$ along the y -axis. The field at the center of each bundle is $B_0 + B_1$. These bundles merge into a uniform field a distance of the order of a on each side of the y -axis.

The energy of the bundle lying in $-\pi/k \leq y \leq +\pi/k$ is

$$\begin{aligned} E &= \frac{1}{2\mu} \int_{-\infty}^{+\infty} dz \int_{-\pi/k}^{+\pi/k} dy (B_1^2 + B_z^2) \\ &= E_0 + \frac{B_1^2}{2\mu} \int_{-\infty}^{+\infty} dz \int_{-\pi/k}^{+\pi/k} dy \left[\frac{4}{a^2 k^2} \left(\frac{z^2}{a^2} \right) \sin^2 ky + \cos^2 ky \right] \exp(-2 \frac{z^2}{a^2}) \end{aligned}$$

where E_0 is the energy of the homogeneous component of the field. The term first order in B_1 drops out because of the integration over y . We obtain finally the energy due to the bunching as

$$E - E_0 = \frac{\pi^{1/2}}{2^{1/2}} \left(\frac{B_1^2}{2\mu} \right) \frac{a}{k} \left(1 + \frac{2}{a^2 k^2} \right)$$

The energy per unit y is

$$\epsilon = \frac{k}{2\pi} (E - E_0) = \frac{\pi^{1/2}}{2^{1/2}} \left(\frac{B_1^2}{2\mu} \right) a \left(1 + \frac{2}{a^2 k^2} \right) \quad (23)$$

For given values of a and B_1 , ϵ is a monotonically decreasing function of k . This demonstrates the tendency for flux tubes to split up where the tube changes size abruptly.

The magnetic stresses at the surface of the sun are of the order of 0.2 the gas pressure; we conclude that the flux tube has sufficient potential energy to carry out the branching, pushing the gas out of its way as it does so. Therefore, at the surface of the sun we expect to find after a time, not one, but several flux tubes, each producing a small spot: Initially the field from the spot was of the general configuration shown in figure 2(b); branching results in something like figure 2(c).

Each branch of a flux tube in a spot group extends of the order of 10^4 km or more down into the convective zone, and so is pushed around by the convective motions there. We would expect the portion of the branch above the surface of the sun to show some of this random motion, with the result that the spots of a group spread out from their initial position. Besides this branching process, one would expect that the magnetic bouyancy, which was initially successful in heaving a region of relatively intense field up to the surface, will continue to operate, though more slowly than at first, to pull up more of the toroidal field. The process will not go far because, as was shown in the first section, the magnetic bouyancy becomes unimportant as one goes to weaker fields and deeper layers. But insofar as the process operates, it should result in a progressively larger region of the toroidal field rising to the surface, with a subsequent increase in the separation of the two parts of the spot group. We conclude from the two effects discussed in this paragraph that each half of a spot group should slowly diverge within itself and from the other half.

The eventual expiration of a given spot group follows from the fact that a flux tube rising from the convective zone up to the photosphere and descending again to the convective zone, to form an inverted "U", does not constitute a regenerative portion of the solar hydromagnetic dynamo; in this prodigal state it dies from diffusion. We must remember that it is not the molecular diffusion, which is negligible, but the eddy diffusivity of the convective zone that is responsible for the decay of the spot; vertical convective velocities of 10 m/sec in the region of the convective zone under the spot give a decay time of only a few months.

4. Motion above the Surface of the Sun.

In this section we shall discuss some of the mechanisms operating in the sun which result in conspicuous motion of matter in the flux tubes above the level of the photosphere. We shall assume that the motions in the convective zone constitute the prime mover, communicating motion directly or indirectly to the levels above the photosphere. The extreme stratification of the solar atmosphere above the reversing layer results in hydrodynamic stability, preventing the communication of motions to the higher levels by other than hydromagnetic mechanisms; we shall assume that in the solar atmosphere a number of flux tubes extend, as a consequence of the magnetic bouyancy, from the convective zone into the regions above the photosphere.

The first mechanism that comes to mind is the relatively slow horizontal or transverse migration of flux tubes, which was discussed in the dissipation of sunspots, and which arises from the fact that the portion of the flux tube down in the convective zone is pushed around by the convective motions there; the portion of the tube above the convective zone follows the horizontal motion of its lower parts. It follows that the observed horizontal motions will not exceed the horizontal convective velocities of, say, 10 m/sec.

The other major class of motions is longitudinal in nature, involving a flow of gas along the magnetic field. Both observation and theory lead us to expect the longitudinal motions to be more violent than the transverse.

Consider a flux tube which for one reason or another is reduced in diameter by an amount δw over a segment of length L . Let the dilatation be uniform over the segment so that

$$\delta w = a w \quad (24)$$

where a is a numerical constant. Assuming that the density is unchanged by the constriction, the mass squeezed out of the segment must be

$$\delta M = \int_0^L ds \rho_i \delta(w^2) = 2a \int_0^L ds \rho_i w^2 \quad (25)$$

Assume that δM issues from only one end of the segment, say $s = L$, where the cross sectional area of the tube is w_L^2 and the density is ρ_{iL} . The distance l that the fluid must move at $s = L$ is

$$l = \frac{\delta M}{\rho_{iL} w_L^2} = 2a \int_0^L ds \left(\frac{\rho_i}{\rho_{iL}} \right) \left(\frac{w}{w_L} \right)^2 \quad (26)$$

Consider the case where the flux tube is in equilibrium with its surroundings. We assume that $p_i \ll p_e$ so that $\rho_i \approx \rho_e$. Using this approximation together with the result derived in (16) that $w \propto p_e^{-1/4}$, we may rewrite (26) as

$$l \approx 2a \int_0^L ds \left(\frac{p_e}{p_{eL}} \right)^{1/2} \left(\frac{T_{eL}}{T_e} \right) \quad (27)$$

If the flux tube is horizontal, then $p_e = p_{eL}$ and $T_e = T_{eL}$, giving $l = 2aL$. If, on the other hand, the tube is vertical with the open end up, l may be considerably greater than $2aL$. For an adiabatic atmosphere $p_e \propto T_e^{\gamma/(\gamma-1)}$, and

$$l \approx 2a \int_0^L ds \left(\frac{T_e}{T_{eL}} \right)^{(2-\gamma)/[2(\gamma-1)]} \quad (28)$$

In the sun T_e may be an order of magnitude greater at the lower end than T_{eL} at the upper end, giving $l > 2aL$.

The amplification of motion is essentially $l/\delta w$: This is $2L/w$ for a horizontal tube and more for a vertical tube open at the upper end. Thus, in a slender flux tube the longitudinal motion may be several orders of magnitude greater than the transverse motion constricting the tube.

The discussion in section 3 indicates that the flux tube forming a sunspot is not in thermal equilibrium with its surroundings as a consequence of the cooling effect. The result is that the tube is of very small diameter at the spot, and the amplification processes discussed in the above paragraph become even more severe. Returning to (26) we write

$$w^2 = w_1^2 (B_1/B)$$

for the flux tube and obtain

$$l = 2a \int_0^L ds \left(\frac{\rho_i}{\rho_1} \right) \left(\frac{B_1}{B} \right) \quad (29)$$

Regarding the spot as the open upper end of the tube, we see that the density ρ_1 in the spot is very much less than in the 10^4 km or more below the spot. On the other hand, B_1 in the spot is considerably greater than below the spot. To form a rough estimate of l , we let $\rho_i \sim 10^3 \rho_1$ and $B \sim 0.1 B_1$. We obtain

$$l \sim 2 \times 10^3 a L$$

from (29). The amplification is

$$l/\delta w \sim 2 \times 10^3 L/w \quad (30)$$

using (24). Even if w is of the same order as L , the amplification of the small constrictive motion of the flux tube below the sunspot

is of the order of 10^3 : 10 m/sec in the convective zone results in 10 km/sec in prominences above the spot. If $w = 0.1 L$, we obtain motions of 100 km/sec, suggesting surge prominences. Comparing (30) with the amplification $2L/w$ for a flux tube in thermal equilibrium with its surroundings, we see that we expect to find the most violent prominence motions associated with the upper end of cooled flux tubes, i.e. with sunspots.

As opposed to motions above the photosphere produced by motions in the convective zone, consider the longitudinal flow of gas within a flux tube arising from a relatively slow change of the equilibrium state of the flux tube: A flare occurring in a spot group may introduce a heating of the portion of the flux tube over the group; the cooling effect in a flux tube discussed in section 3 results in large changes in the static equilibrium configuration.

It is with the latter effect that we wish to dwell for the moment: It was shown in section 3 how the integrated effect of a small change in $\rho_i - \rho_e$ may profoundly change the volume of a flux tube below a sunspot; the change is illustrated between figures 2(a) and 2(b).

Quantitatively, the mass per unit length of a vertical flux tube at time t is

$$dM(z,t)/dz = w^2(z,t) \rho_i(z,t)$$

But from (2) and the fact that $B \propto w^{-2}$ we may write that

$$w^2(z,t) = w^2(0,t) [p_m(0,t)/p_m(z,t)]^{1/2}$$

Thus

$$M(z,t) = w'(0,t) \int_0^z dz \rho_i(z,t) \left[\frac{p_m(0,t)}{p_m(z,t)} \right]^{1/2} \quad (31)$$

If the flux tube is in thermal equilibrium with its surroundings at $t = 0$, with the corresponding divergent configuration shown in figure 2(a), and at some later time t is in the converging configuration, resulting from the cooling effect, shown in figure 2(b), then the mass of the flux tube up to the photosphere at $z = L$ has decreased by the amount

$$M(L,0) - M(L,t) = \int_0^L dz \left\{ w'(0,0) \rho_i(z,0) \left[\frac{p_m(0,0)}{p_m(z,0)} \right]^{1/2} - w'(0,t) \rho_i(z,t) \left[\frac{p_m(0,t)}{p_m(z,t)} \right]^{1/2} \right\} \quad (32)$$

Now $\rho_i(z,0) \cong \rho_i(z,t)$. Except near the base of the tube $p_m(z,t) \gg p_m(0,t)$ and $p_m(z,0) \ll p_m(0,0)$. Thus, if we assume that $w(0,0)$ and $w(0,t)$ are comparable, (31) reduces to the order of magnitude relation

$$M(L,0) - M(L,t) \sim w'(0,0) \int_0^L dz \rho_i(z,0) \left[\frac{p_m(0,0)}{p_m(z,0)} \right]^{1/2} = M(L,0) \quad (33)$$

It follows that a large fraction of the gas in the flux tube beneath the spot, at $z = L$, has flowed away. Much of the gas will issue through the lower end of the tube, but some will undoubtedly flow out of the spot at the upper end, inflating the flux tube over the spot. Insofar as the cooling and subsequent converging of the diffuse portions of the flux tube below the spot is a continuous process, going on throughout the life of the spot group, one may expect to find a small continuous efflux of matter from the spot. It is possible that the Evershed effect is the observation of just this efflux.

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Appendix I. Kinematics of a Flux Tube.

To supplement the rather brief discussion in section 2 of the equilibrium of a flux tube let us consider the stresses within a tube of flux which may not be in equilibrium. Let us idealize the flux tube to be of square cross section of side w and to contain a magnetic field B uniform over the cross section. Let us confine the tube to the yz -plane. We assume a gravitational field of acceleration g in the negative z direction. We let s represent distance measured along the axis of the tube from left to right, and θ the inclination of the tube. We shall take the tube to be sufficiently slender compared to the characteristic lengths of the medium in which it is suspended that

$$dw/ds \ll 1, \quad d\theta/ds \ll 1/w. \quad (34)$$

Let p_i, ρ_i, T_i and p_e, ρ_e, T_e represent the state of the material medium inside and outside the flux tube, respectively. Assuming the molecular weight of the medium to be uniform throughout, we write

$$p_i = (k/m) T_i \rho_i, \quad p_e = (k/m) T_e \rho_e. \quad (35)$$

We shall assume that (1) is satisfied, which puts the flux tube in local lateral equilibrium. Finally, we shall assume that the medium outside is in equilibrium, satisfying the barometric relation

$$\partial p_e / \partial z = -\rho_e g, \quad \partial p_e / \partial x = \partial p_e / \partial y = 0 \quad (36)$$

Consider the element $w^2 ds$ of the flux tube shown in elevation and plan view in figures 3(a) and 3(b) respectively.

The angle between the sides and the axis of the flux tube is

$$\alpha = (1/2) dw/ds \quad (37)$$

Consider the force $F_s ds$ in the s-direction on the element. The weight of the element is $g\rho_i w^2 ds$. On the left hand end there is a net pressure $p_i - B^2/(2\mu)$ on the area w^2 ; a corresponding force is exerted on the opposite end of the element. The sides of the element parallel to the yz-plane experience a pressure p_e inclined at an angle $(\pi/2 - \alpha)$ to the axis of the element. Finally, the remaining pair of sides experience pressures $p_e \pm (1/2)(\partial p_e/\partial z)w \cos \theta$. Multiplying the above pressures by their respective areas and taking the s component, we obtain

$$\begin{aligned} F_s ds = & -g\rho_i w^2 \sin \theta ds + 2p_e w \alpha ds \\ & + (p_i - \frac{B^2}{2\mu}) w^2 - \left[(p_i - \frac{B^2}{2\mu}) + \frac{\partial}{\partial s} (p_i - \frac{B^2}{2\mu}) ds \right] (w^2 + \frac{\partial w^2}{\partial s} ds) \\ & + (p_e + \frac{1}{2} \frac{\partial p_e}{\partial z} w \cos \theta) w \left(1 - \frac{w}{2} \frac{d\theta}{ds} \right) \alpha ds \\ & + (p_e - \frac{1}{2} \frac{\partial p_e}{\partial z} w \cos \theta) w \left(1 + \frac{w}{2} \frac{d\theta}{ds} \right) \alpha ds \end{aligned} \quad (38)$$

Using (1) to eliminate p_i , (36) to eliminate $\partial p_e/\partial z$, and (37) to eliminate α , we may simplify (38) to

$$F_s = \frac{d}{ds} \left(w^2 \frac{B^2}{2\mu} \right) + g w^2 \sin \theta (\rho_e - \rho_i) \quad (39)$$

neglecting terms of second and higher order in dw/ds and $d\theta/ds$.

Since we are considering a flux tube, it follows that the total flux Φ is independent of s. Then

$$B = \Phi / w^2 \quad (40)$$

and

$$\frac{d}{ds} \left(w^2 \frac{B^2}{2\mu} \right) = \frac{\Phi^2}{\mu} \frac{d}{ds} \left(\frac{1}{w^2} \right) = \frac{\Phi^2}{2\mu} w^2 \frac{d}{ds} \left(\frac{1}{w^4} \right) = w^2 \frac{d}{ds} \frac{B^2}{2\mu}$$

Thus we may write (39) as

$$F_s = w^2 \left[\frac{d}{ds} \left(\frac{B^2}{2\mu} \right) + g \sin \theta (\rho_e - \rho_i) \right] \quad (41)$$

Consider the forces normal to the axis of the tube.

Similar to (38) we write

$$\begin{aligned} F_n ds = & -g\rho_i w^2 \cos \theta ds - 2 \left(\rho_i - \frac{B^2}{2\mu} \right) w^2 \frac{1}{2} \frac{d\theta}{ds} ds \\ & - \left(\rho_e + \frac{1}{2} \frac{\partial \rho_e}{\partial z} w \cos \theta \right) w \left(1 - \frac{w}{2} \frac{d\theta}{ds} \right) ds + \left(\rho_e - \frac{1}{2} \frac{\partial \rho_e}{\partial z} w \cos \theta \right) w \left(1 - \frac{w}{2} \frac{d\theta}{ds} \right) ds, \end{aligned}$$

which reduces to

$$F_n = w^2 \left[\frac{B^2}{\mu} \frac{d\theta}{ds} + g \cos \theta (\rho_e - \rho_i) \right] \quad (42)$$

Consider the special case where the tube is in longitudinal equilibrium. Then $F_s = 0$, and (41) reduces to

$$0 = \frac{d}{ds} \left(\frac{B^2}{2\mu} \right) + g \sin \theta (\rho_e - \rho_i) \quad (43)$$

Now

$$\frac{d}{ds} = \frac{dz}{ds} \frac{d}{dz} = \sin \theta \frac{d}{dz} \quad (44)$$

Hence, for any inclination of the tube we obtain

$$0 = \frac{d}{dz} \left(\frac{B^2}{2\mu} \right) + g (\rho_e - \rho_i), \quad (45)$$

which was obtained in (14) for a vertical tube. If we differentiate (1) with respect to z and use (36), we obtain

$$0 = \frac{d}{dz} \left(\frac{B^2}{2\mu} \right) + g\rho_e + \frac{d\rho_e}{dz} \quad (46)$$

Comparing (45) and (46) we find that equilibrium requires that ρ_i obey the barometric law

$$\frac{dp_i}{dz} = -\rho_i g \quad (47)$$

The physically obvious fact that a flux tube is stable against a local constriction or expansion is readily demonstrated from (41). Consider a horizontal flux tube of uniform cross section. Let us pinch the tube over a finite extent of its length. B is increased in the restriction. Approaching the restriction from the left we have $(d/ds)(B'/u) > 0$. (41) gives $F_s > 0$, causing the fluid to flow into the restricted region, and restoring the tube to its initial uniformity.

Appendix II. Flux Tube in Thermal Equilibrium

Consider a flux tube in thermal equilibrium with its surroundings. The static equilibrium of such a tube was investigated briefly in section 2. There we found that $B \propto p_e^{1/2}$, so that $p_i \propto p_e$. From (1) it follows that $\rho_i \propto p_e$. Since $B w^2$ is the total flux through the tube and is constant, we have that $w \propto p_e^{-1/4}$. We note that the mass per unit length $w^2 \rho_i$ decreases with height for a tube in either an isothermal or an adiabatic atmosphere.

We shall now inquire into the longitudinal motions within a flux tube and the variation of the magnetic bouyancy as a segment of an initially horizontal flux tube is displaced vertically by the magnetic bouyancy. We would like to know whether the tube can be expected to tend toward the static equilibrium configuration discussed in the above paragraph and whether the magnetic bouyancy continues to function even after large displacement; the fact that $w^2 \rho_i$ decreases with height implies that there must be a longitudinal flow away from the elevated portion of the tube. We shall find just such a flow from the following calculations; the flow allows an approach to $B \propto p_e^{1/2}$ and guarantees that the magnetic bouyancy will not fail after some finite displacement.

Consider how conditions will vary when a flux tube initially horizontal and of uniform cross section is displaced vertically by some small but varying amount $\delta z(y)$. We are particularly interested in finding whether an upward bulging of the tube will result in fluid flowing along the tube away from the bulge or toward the bulge. Thus, we shall assume that $T_i = T_e = T$

and constrain the fluid within the tube to move only in the z -direction, allowing no flow along the tube; we then investigate the longitudinal force F_z to see which way along the tube the fluid would flow if the constraints were removed. The tube will have to be held in place, of course, by external forces because of the magnetic bouyancy and longitudinal stresses.

We shall assume that $d\delta z/ds \ll 1$. Then

$$\sin\theta \cong d\delta z/ds, \quad ds \cong dy. \quad (48)$$

Using (5), (40), and (46), (39) may be written

$$F_z = \frac{\Phi^2}{\mu w^2} \frac{d\delta z}{dy} \left[\frac{mg}{2kT} - \frac{2}{w} \frac{dw}{d\delta z} \right]$$

Consider $(2/w)(dw/d\delta z)$. With the constraint that there be no longitudinal flow within the tube, it follows that

$$\rho = \rho_0 (w_0/w)^2, \quad B = B_0 (w_0/w)^2 \quad (49)$$

Then (5) may be written

$$\rho_0 (w/w_0)^4 - \rho_0 (w/w_0)^2 - m B_0^2 / (2\mu kT) = 0 \quad (50)$$

Because we are considering only a small vertical displacement $\delta z(y)$, we may introduce the approximation

$$\left. \begin{aligned} \left(\frac{w}{w_0}\right)^2 &= 1 + \left[\frac{d}{dz} \left(\frac{w}{w_0}\right)^2\right]_0 \delta z + O^2(\delta z), \quad T = T_0 + \left(\frac{dT}{dz}\right)_0 \delta z + O^2(\delta z) \\ \rho &= \rho_0 + \left(\frac{d\rho}{dz}\right)_0 \delta z + O^2(\delta z) \end{aligned} \right\} \quad (51)$$

Solving (35) for ρ , differentiating with respect to z , dividing by ρ , and using (36) to eliminate $\partial\rho/\partial z$, we obtain

$$\frac{1}{\rho} \frac{d\rho}{dz} = - \frac{mg}{kT_0} - \frac{1}{T_0} \frac{dT_0}{dz} \quad (52)$$

Putting (51) and (52) into (50), we obtain finally

$$\left. \begin{aligned} \frac{d}{d\delta z} \left(\frac{w}{w_0} \right)^2 &= \left[\frac{mg}{kT} + \frac{1}{T} \frac{dT}{dz} \left(1 - \frac{p_{\infty}}{p_0} \right) \right] \left(1 + \frac{p_{\infty}}{p_0} \right)^{-1} + O(\delta z) \\ &= \left[\frac{mg}{kT} p_{\infty} + \frac{1}{T} \frac{dT}{dz} p_0 \right] (p_{\infty} + p_{\infty})^{-1} + O(\delta z) \end{aligned} \right\} \quad (53)$$

But

$$(z/w)(dw/d\delta z) = (1/w_0^2)(dw^2/d\delta z) + O(\delta z)$$

Thus, to the degree of approximation used in (51), (53) gives $(z/w)(dw/d\delta z)$; the equation for F_z may be rewritten as

$$F_z = \frac{d\delta z}{dy} \frac{\Phi^2}{\mu w^2} \left\{ \frac{mg}{2kT} - \left[\frac{mg}{kT} p_{\infty} + \frac{1}{T} \frac{dT}{dz} p_0 \right] (p_{\infty} + p_{\infty})^{-1} \right. \quad (54)$$

$$\left. + O'(\delta z) \right\}$$

If the medium through which the flux tube passes is of uniform temperature so that dT/dz vanishes, then

$$F_z = - \frac{d\delta z}{dy} \frac{\Phi^2}{\mu w^2} \frac{mg}{2kT} \frac{p_{\infty} - p_{\infty}}{p_{\infty} + p_{\infty}} \quad (55)$$

If, on the other hand, it is an adiabatic atmosphere, then

$$\frac{1}{T} \frac{dT}{dz} = \frac{\gamma-1}{\gamma} \frac{1}{p} \frac{dp}{dz} = - \frac{(\gamma-1)}{\gamma} \frac{mg}{kT}$$

and

$$F_z = - \frac{d\delta z}{dy} \frac{\Phi^2}{\mu w^2} \left(\frac{\gamma-1}{\gamma} \right) \left(\frac{mg}{2kT} \right) \left(\frac{p_{\infty} - p_{\infty}}{p_{\infty} + p_{\infty}} \right) \quad (56)$$

We see from (55) and (56) that for both an isothermal and an adiabatic atmosphere $d\delta z/dy > 0$ implies that $F_z < 0$, resulting in a flow of fluid out of the raised portions of the flux tube. This has the effect of increasing p_{∞} relative to p_0 and enhances the magnetic buoyancy in the upward bulges. Only when dT/dz is sufficiently negative to reverse the sign of F_z

will the longitudinal flow degenerate the magnetic bouyancy; certainly in any large-scale region T never decreases at a rate significantly greater than the adiabatic rate because of the extreme convective instability that would result from a more rapid decrease.

Appendix III. Adiabatic Flux Tube.

Consider a flux with an adiabatic interior. T_i no longer need be equal to T_e , and many physical relations which are taken for granted in Appendix II are no longer obvious. We have in mind the same questions as when $T_i = T_e$, viz. whether a flux tube displaced vertically by the magnetic bouyancy tends toward the static equilibrium configuration, and whether the magnetic bouyancy vanishes after some finite displacement. The latter question no longer has the unambiguous answer it had when $T_i = T_e$.

Consider the static equilibrium of a flux tube with an adiabatic interior; the relation between p_i and ρ_i is accordingly

$$p_i = p_{i0} (\rho_i / \rho_{i0})^\gamma \quad (57)$$

If the flux tube is in equilibrium, we may combine (57) with (47). We obtain the familiar relations for an adiabatic atmosphere

$$\rho_i = \rho_{i0} \left[1 - \frac{\gamma-1}{\gamma} \frac{mg}{kT_{i0}} z \right]^{\gamma/(\gamma-1)}, \quad p_i = p_{i0} \left[1 - \frac{\gamma-1}{\gamma} \frac{mg}{kT_{i0}} z \right]^{\gamma/(\gamma-1)} \quad (58)$$

ρ_{i0} , p_{i0} , and T_{i0} are the values of ρ_i , p_i , and T_i at $z = 0$. Using (1), the magnetic pressure for static equilibrium is

$$p_m = p - p_i = p_e - p_{i0} \left[1 - \frac{\gamma-1}{\gamma} \frac{mg}{kT_{i0}} z \right]^{\gamma/(\gamma-1)} \quad (59)$$

The width w of the tube may be computed from the fact that it varies as $p_m^{-1/4}$.

Consider the special case that the atmosphere outside the tube is an adiabatic atmosphere and further that $T_i = T_e$ at $z = 0$. Now independently of the latter condition, T_i and T_e vary linearly with height according to

$$T_i = T_{i0} \left[1 - \frac{\gamma-1}{\gamma} \frac{mg}{kT_{i0}} z \right], \quad T_e = T_{e0} \left[1 - \frac{\gamma-1}{\gamma} \frac{mg}{kT_{e0}} z \right]$$

Thus, setting $T_i = T_e$ at $z = 0$ implies that $T_i = T_e$ at all heights in the atmosphere; the tube is in thermal equilibrium with its surroundings, even though insulated from them, and the relations worked out in the previous section for thermal equilibrium in an adiabatic atmosphere are valid.

The magnetic bouyancy depends on the sign of $\rho_i - \rho_e$ which must be investigated quantitatively because, in an adiabatic atmosphere, it depends critically on the initial conditions. Consider, then a long horizontal uniform flux tube. There will be no longitudinal flow under these conditions and $B \propto \rho_i$. Thus

$$p_i = p_{i0} (\rho_i / \rho_{i0})^2 \quad (60)$$

The zero subscript denotes initial values. Comparing (60) with (57) it follows that p_i varies more rapidly with the density than does ρ_i since $\gamma < 2$. A vertical displacement of the flux tube, with the resulting change in p_e , results in p_i taking up more than its share of the pressure change. Thus p_i does not have to vary as rapidly, and we conclude that

$$(1/\rho_i)(d\rho_i/dz) < (1/p_e)(dp_e/dz) \quad (61)$$

The slower rate of variation of p_i may be deduced quantitatively from (1). Differentiating with respect to z , we obtain

$$\frac{dp_i}{dz} = \frac{dp_e}{dz} - \frac{B'}{B} \frac{1}{B} \frac{dB}{dz} \quad (62)$$

But from (49) and (58) it follows that

$$(1/B)(dB/dz) = (1/\rho_i)(d\rho_i/dz) = (1/\gamma)(1/\rho_i)(dp_i/dz) \quad (63)$$

Putting (63) into (62) and solving for $(1/\rho_i)(dp_i/dz)$, we obtain

$$\frac{1}{\rho_i} \frac{dp_i}{dz} = \frac{dp_e}{dz} \left[\rho_i + \frac{B^2}{\gamma \mu} \right]^{-1} = \frac{1}{\rho_e} \frac{dp_e}{dz} \left[1 + \frac{2-\gamma}{\gamma} \frac{p_m}{p_e} \right]^{-1} \quad (64)$$

Thus, with $\gamma < 2$ we obtain (61).

If the medium outside the flux tube varies adiabatically with height z then, besides (61), we have

$$(1/\rho_i)(d\rho_i/dz) < (1/\rho_e)(d\rho_e/dz), (1/T_i)(dT_i/dz) < (1/T_e)(dT_e/dz) \quad (65)$$

To obtain relations between ρ_e and ρ_i etc. rather than just their derivatives, we put (57) and (60) into (1), obtaining

$$p_e = p_{i0}(\rho_i/\rho_{i0})^\gamma + p_{m0}(\rho_i/\rho_{i0})^2 \quad (66)$$

(66) gives ρ_i in terms of p_e . p_i , p_m , and w may then be computed using (57), (60), and (49) respectively.

If the external medium is an adiabatic atmosphere, then

$$p_e = p_{e0}(\rho_e/\rho_{e0})^\gamma \quad (67)$$

and (66) may be written as

$$p_{e0}(\rho_e/\rho_{e0})^\gamma = p_{i0}(\rho_i/\rho_{i0})^\gamma + p_{m0}(\rho_i/\rho_{i0})^2 \quad (68)$$

Consider how ρ_i and ρ_e compare near the top of the atmosphere where ρ_e approaches zero. ρ_i also approaches zero, so that $\rho_i/\rho_{i0} \ll 1$. (68) reduces to

$$p_{e0}(\rho_e/\rho_{e0})^\gamma = p_{i0}(\rho_i/\rho_{i0})^\gamma$$

Thus

$$\rho_e/\rho_i = (\rho_{e0}/\rho_{i0})(p_i/p_e)^{1/\gamma} = (\rho_{e0}/\rho_{i0})^{1-\gamma/\gamma} (T_{i0}/T_{e0})^{1/\gamma} \quad (69)$$

For the special case that $T_i = T_e$, we have that $\rho_e > \rho_i$; (69) implies that $\rho_e/\rho_i > 1$ as ρ_e approaches zero. If, on the otherhand, $\rho_e = \rho_i$, then $T_e > T_i$, and (69) tells us that $\rho_e/\rho_i < 1$, which also follows from (65). Thus, we have demonstrated that, beginning with a flux tube initially in thermal equilibrium with its surroundings, the magnetic bouyancy is operative at all heights in an adiabatic atmosphere to which the flux tube may be displaced; the presence of flux tubes induces convection in an adiabatic atmosphere. If, on the other hand, the flux tube initially satisfies (1) by virtue of $T_i < T_e$ rather than $\rho_i < \rho_e$, then the flux tube is stable against vertical displacement; the presence of flux tubes inhibits convection in an adiabatic atmosphere.

The critical condition giving an initial magnetic bouyancy which vanishes as $\rho_e/\rho_i \rightarrow 1$ at the top of the atmosphere may be obtained from (69) by putting $\rho_e = \rho_i$. We obtain the relation

$$\rho_i = \rho_e (T_i/T_e)^{\gamma(\gamma-1)} \quad (70)$$

This is the condition that the variation of the matter within the flux tube is adiabatic during the generation of the magnetic field.

To determine ρ_e/ρ_i as $\rho_e \rightarrow 0$ we note from (57) and (60) that ρ_e decreases more rapidly with decreasing ρ_i than does ρ_i . Thus ρ_e/ρ_i approaches zero as the top of the atmosphere is approached and (1) becomes

$$\rho_e/\rho_i = 1 + \rho_e/\rho_i \sim 1 \quad (71)$$

To compute T_e/T_i as $\rho_e \rightarrow 0$ we use (35), (69) and (71). We find

$$T_i / T_e = (T_e / T_{i0})^{1/\gamma} (\rho_{i0} / \rho_e)^{(\gamma-1)/\gamma} \quad (72)$$

If initially $T_{e0} = T_{i0}$, we have $\rho_{i0} < \rho_{e0}$ and conclude from (72) that $T_i > T_e$; the interior of the flux tube becomes hotter than its surroundings, which also follows from (65). If, on the other hand, $\rho_{i0} = \rho_{e0}$, then $T_i < T_{e0}$, and we conclude that $T_i < T_e$. Again the critical case leading to $T_i = T_e$ at the top of the atmosphere yields the adiabatic relation (70).

As in the previous section where $T_i = T_e$, let us investigate F_z as a result of a small vertical adiabatic displacement of an initially horizontal and uniform flux tube in thermal equilibrium with its surroundings. We shall again introduce the constraint that there be no longitudinal flow; the resulting F_z will tell us whether the fluid would flow toward or away from an upward bulge of the tube if the constraint were removed.

From (39), (40), and (48) we obtain

$$F_z = \frac{d\delta z}{ds} \left[\frac{\Phi^2}{4\mu w_0^2} \frac{1}{\rho_{i0}} \frac{d\rho_i}{d\delta z} + g w^2 (\rho_e - \rho_i) \right] \quad (73)$$

We shall compute the quantity in brackets omitting terms $O(\delta z)$, and obtain F_z omitting terms $O^2(\delta z)$. Thus, using initial values, we obtain from (5) that

$$w^2 (\rho_e - \rho_i) = w_0^2 (\rho_{e0} - \rho_{i0}) + O(\delta z) = \frac{mg}{kT_0} \left(\frac{\Phi^2}{2\mu w_0^2} \right) + O(\delta z) \quad (74)$$

Since the fluid inside the flux tube varies adiabatically, we obtain

$$\frac{1}{\rho_i} \frac{d\rho_i}{dz} = \frac{1}{\gamma} \frac{1}{\rho_i} \frac{d\rho_i}{dz} = \frac{1}{\gamma} \frac{1}{\rho_e} \frac{d\rho_e}{dz} \left[1 + \frac{2-\gamma}{\gamma} \frac{p_e}{\rho_e} \right]^{-1}$$

the latter by (64). Using (35) and (36) we obtain finally

$$\frac{1}{\rho_0} \frac{d\rho_0}{dz} = -\frac{1}{\gamma} \frac{mg}{kT_0} \left[1 + \frac{2-\gamma}{\gamma} \frac{p_m}{p_0} \right]^{-1} + O(\delta z) \quad (75)$$

Substituting (74) and (75) into (73), we obtain

$$F_s = -\frac{d\delta z}{ds} \left(\frac{mg}{kT_0} \right) \left(\frac{\Phi^2}{2\mu_0 w^2} \right) \left(\frac{\gamma-2}{\gamma} \right) \frac{p_{00} - p_{m0}}{p_{00} + (2-\gamma/\gamma)p_m} + O'(\delta z) \quad (76)$$

which is to be compared with (56). $d\delta z/ds > 0$ implies that $F_s < 0$, indicating a longitudinal flow along the tube away from an upward bulge, as in the case worked out previously where $T_0 = T_1$. The transport of matter from the raised, and therefore expanded, portion of the tube enhances p_m relative to p_0 by removing part of the fluid producing p_0 and compressing B. This enhances the magnetic bouyancy and explains why ρ/ρ_0 is greater than unity, as shown by (69), at the top of the atmosphere, even though p_m , on which the magnetic bouyancy depends, drops off with ρ_0 more rapidly than does p_0 .

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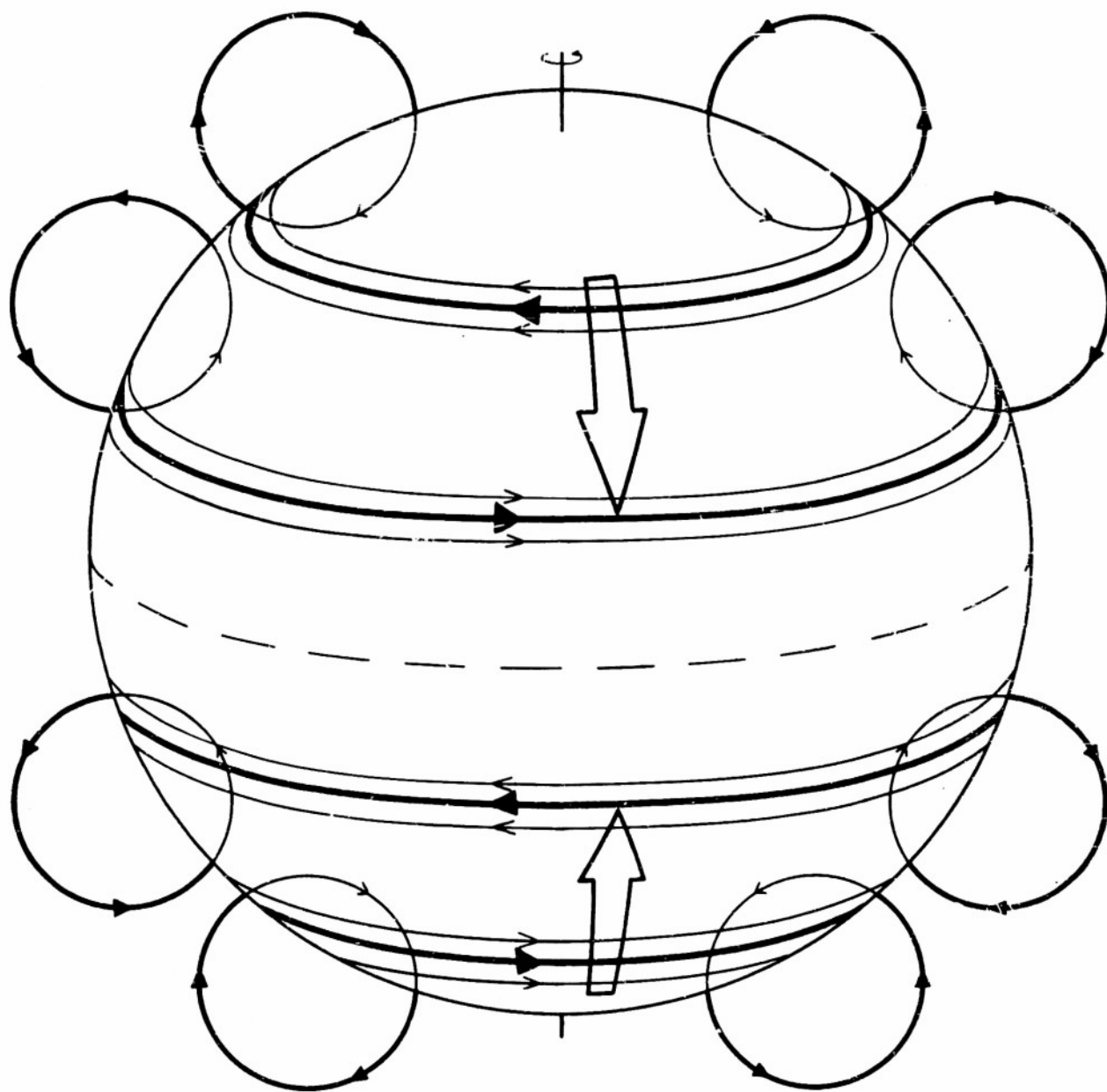


Figure 1

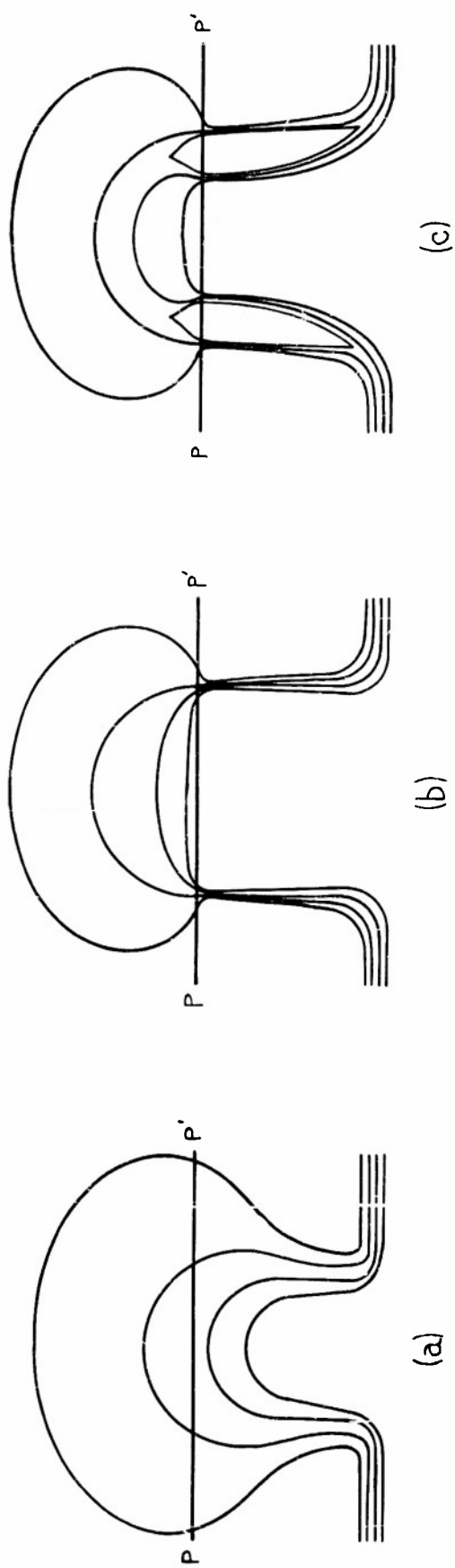
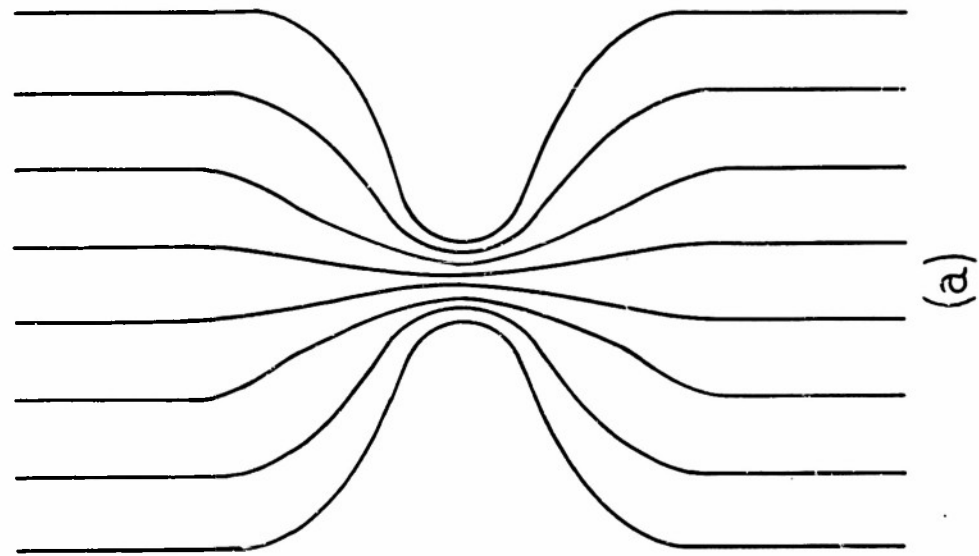
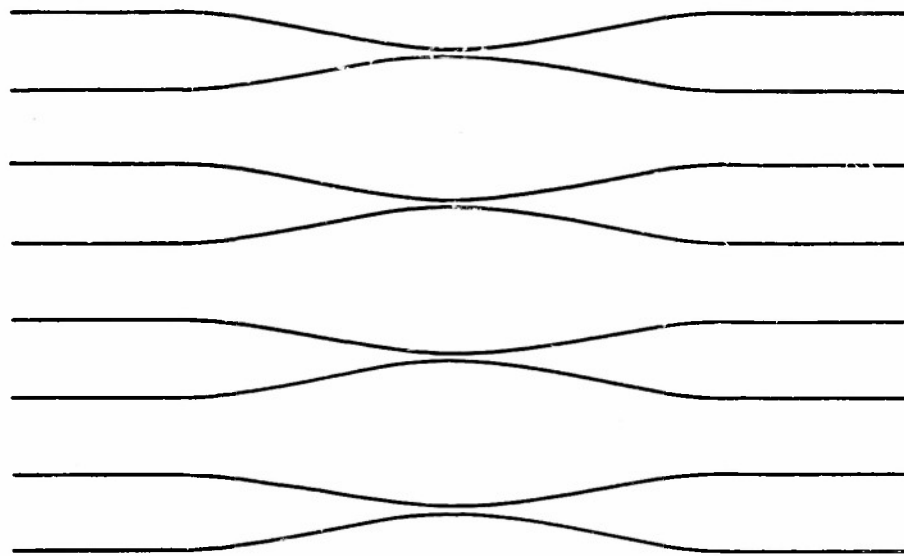


Figure 2



(a)



(b)

Figure 3

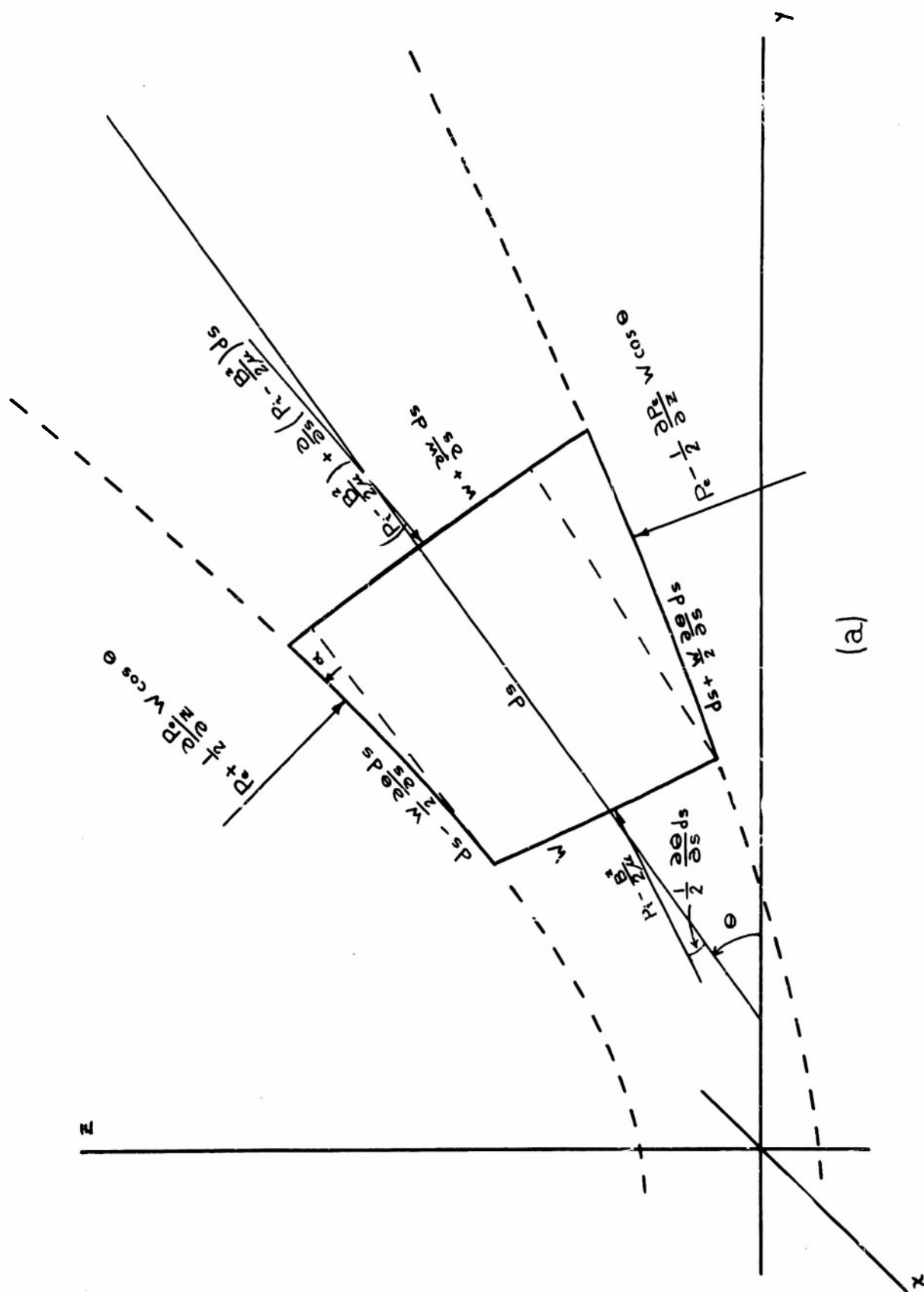


Figure 4

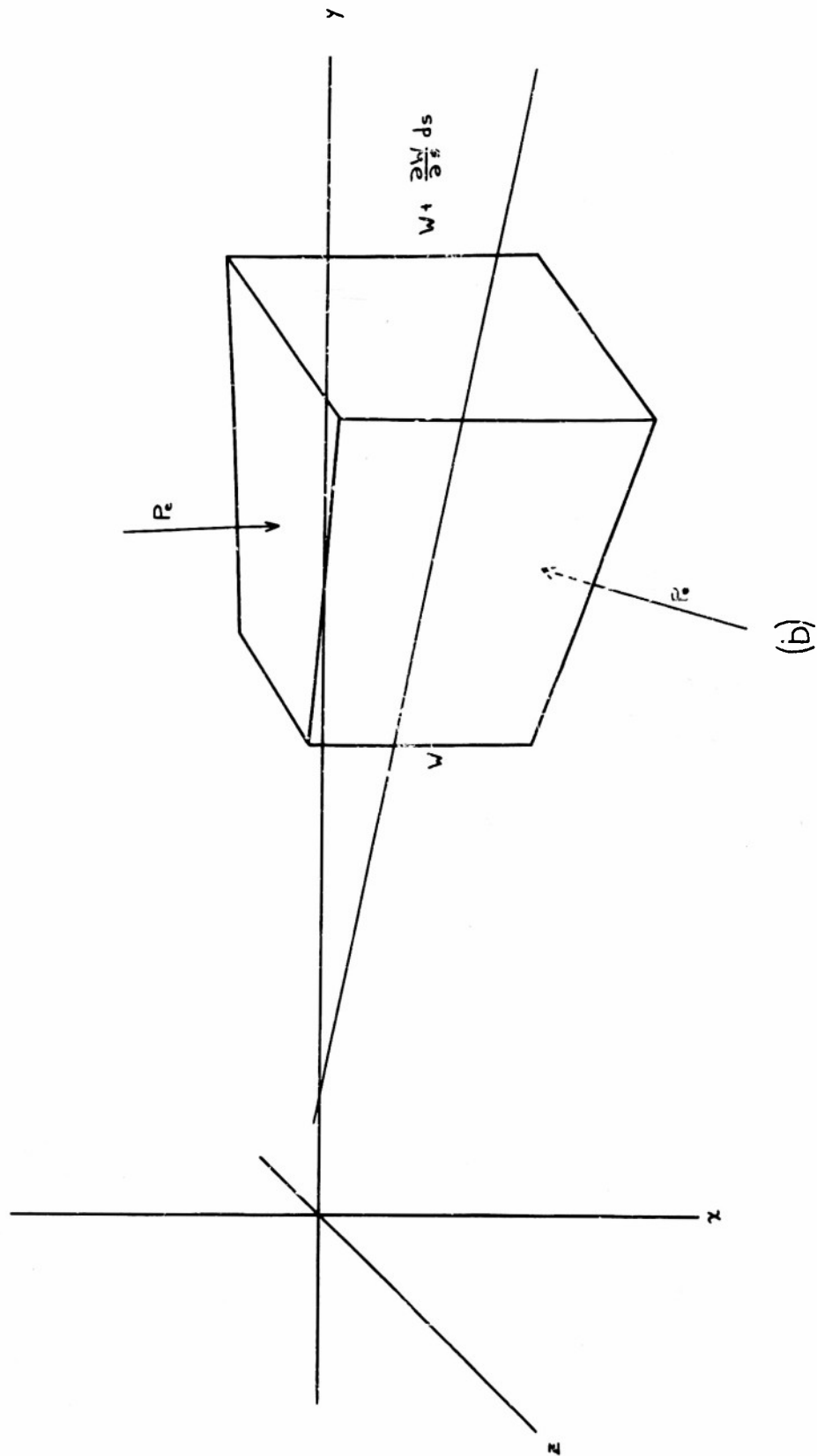


Figure 4

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